Appendix 3: Math Review

Base 10 is composed of 10 integers. These are 0,1,2,3,4,5,6,7,8, and 9. Using these 10 integers, any number can be expressed quite easily. Base 10 is called base 10 because it uses 10 unique digits to express numbers. Most math problems use base 10.

Base 2 is composed of 2 integers. These are 1 and 0. Just like with base 10, any number can be expressed in base 2. Computers use base 2 for mathematical calculations. The following table shows how to count from 0 to 7 in both systems.

Base 10	Base 2
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Base 2 is very useful for information theory because each digit in base 2 contains exactly 1 bit of information. So when information was defined, engineers naturally decided to define it in terms of base 2.

Exponents

Exponents indicate that a number should be multiplied by itself a certain number of times. In the following equation: $10^3 = 10 \times 10 \times 10 = 1000$, the exponent is 3 and in this equation $2^4 = 2 \times 2 \times 2 \times 2 = 16$, the exponent is 4. Any number multiplies by itself zero times is by definition equal to 1, so $10^0 = 2^0 = 10{,}000^0 = 1$.

Logarithms

The symbol log on a calculator indicates a logarithm. Logarithms are the inverse operation of exponents. For example, if the exponent 3 is used to raise 10 to a power the result is 1,000. If the log function is used to find the logarithm on 1,000, the result is equal to the exponent which in this case is 3.

$$10^3 = 10 \times 10 \times 10 = 1{,}000$$

$$\text{Log}(1{,}000) = 3$$

Logarithms must assume a base. The log function on a calculator assumes that the user wants to calculate the logarithm for base 10. The equation below does not work because the log function is only the inverse when the number 10 is raised to a power by the exponent.

$$2^3 = 2 \times 2 \times 2 = 8$$

$$\text{Log}(8) = 0.9 \neq 3$$

Because information theory uses base 2, the exponents always operate on the number *2*. The log function can be used to find the inverse of this operation if the answer is multiplied by a conversion factor, 3.32. This only works for exponents raising the number *2* to a power.

$$2^3 = 2 \times 2 \times 2 = 8$$

$$3.32 \times \text{Log}(8) = 3$$

Addition with logarithms is the same operation as multiplication. The following equation will multiply 100 x 100 using logarithms.

Log (100) + Log (100) = 2 + 2 = 4

The number *4* is now used to raise the number *10* to a power:

10^4 = 10 x 10 x 10 x 10 = 10,000

Notice that 100 x 100 also equals 10,000. Today with handheld calculators logarithms are seldom used to multiply large numbers, but the property of logarithms that turns multiplication into addition is still very useful for information theory. It is this property of logarithms that led scientists to define information using logarithms because it allows information to be added without having to worry about multiplication.

Consider the following two sentences:

I like the dog.

I like the cat.

Both sentences use 15 characters, 11 letters, 3 spaces, and 1 period. If the sentences are constructed by drawing blocks labeled with the 26 letters in the alphabet, 1 space and 1 period, the odds of drawing either sentence is 1 in 5.1 x 10^{21}. The odds of drawing the first sentence followed by the second are 1 in (5.1 x10^{21}) x (5.1 x 10^{21}) = 1 in 2.6 x 10^{43}. Intuitively, these two sentences together should have twice as much information as either one by itself, and this is where the log function comes into play.

3.32 x Log (5.1 x 10^{21}) = 72 bits for either sentence alone.

3.32 x Log (2.6 x 10^{43}) = 144 bits for both sentences